

**1. STRESS & STRAIN****Stress:**

$$\sigma = \frac{F}{A} \text{ [N/m}^2 = \text{Pa]}$$

**Types of Stress:**

- ▶ **Tensile/Compressive:** Normal force per unit area
- ▶ **Shear (Tangential):** Tangential force per unit area
- ▶ **Bulk (Volume):** Pressure applied uniformly

**Strain (dimensionless):**

$$\text{Longitudinal: } \varepsilon_L = \frac{\Delta L}{L}$$

$$\text{Shear: } \varepsilon_s = \frac{\Delta x}{L} = \tan \theta \approx \theta$$

$$\text{Volumetric: } \varepsilon_V = \frac{\Delta V}{V}$$

**Elastic limit:** Maximum stress beyond which body does not return to original shape.**Plastic behaviour:** Permanent deformation beyond elastic limit.**\* Common Pitfall**

- ▶ Strain is **dimensionless**; no unit
- ▶ Stress has units (Pa); strain does not
- ▶ Within elastic limit: Hooke's Law valid; beyond it: plastic deformation

**2. ELASTIC MODULI****Young's Modulus (Y):**

$$Y = \frac{\text{Tensile Stress}}{\text{Longitudinal Strain}} = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L} \text{ [Pa]}$$

**Bulk Modulus (B):**

$$B = -\frac{P}{\Delta V/V} = -V \frac{\Delta P}{\Delta V} \text{ [Pa]}$$

Negative sign: volume decreases with pressure increase.

**Compressibility:**  $k = \frac{1}{B} \text{ [Pa}^{-1}\text{]}$ **Modulus of Rigidity / Shear Modulus ( $\eta$  or  $G$ ):**

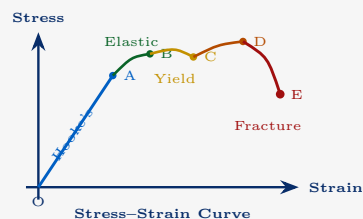
$$\eta = \frac{\text{Shear Stress}}{\text{Shear Strain}} = \frac{F/A}{\theta} \text{ [Pa]}$$

**Typical values:**Steel:  $Y \approx 2 \times 10^{11} \text{ Pa}$ Rubber:  $Y \approx 5 \times 10^6 \text{ Pa}$ Water:  $B \approx 2 \times 10^9 \text{ Pa}$ **\* Common Pitfall**

- ▶  $Y$  is defined only for solids; gases/liquids have no  $Y$
- ▶ Gases have only  $B$  (no  $Y$  or  $\eta$ )
- ▶ Higher  $Y \Rightarrow$  more rigid (less deformation for same force); steel > rubber

**3. HOOKE'S LAW & STRESS-STRAIN CURVE****Hooke's Law:**  $\sigma \propto \varepsilon$ 

$$F = -kx \text{ (for spring/wire)}$$



- ▶ **O→A:** Proportional limit (Hooke's Law)
- ▶ **A→B:** Elastic limit (no permanent deformation)
- ▶ **B→C:** Yield point (permanent deformation begins)
- ▶ **C→D:** Strain hardening
- ▶ **D→E:** Necking  $\rightarrow$  Fracture

**Elastic Potential Energy in wire:**

$$U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times V = \frac{1}{2} \times Y \varepsilon^2 \times V$$

$$\text{Energy density: } u = \frac{U}{V} = \frac{\sigma^2}{2Y} = \frac{1}{2} Y \varepsilon^2$$

**\* Common Pitfall**

- ▶ Elastic limit  $\neq$  proportional limit; elastic limit is slightly beyond proportional limit
- ▶ Breaking stress < Ultimate stress (stress at D) for ductile materials
- ▶ Brittle materials fracture before yield point

**4. POISSON'S RATIO & RELATIONS**

$$\nu = -\frac{\text{Lateral Strain}}{\text{Longitudinal Strain}} = -\frac{\Delta D/D}{\Delta L/L}$$

**Range:**  $-1 \leq \nu \leq 0.5$ Practical range:  $0 < \nu < 0.5$ For rubber:  $\nu \approx 0.5$  (incompressible)For steel:  $\nu \approx 0.3$ **Relations between elastic constants:**

$$Y = 2\eta(1 + \nu)$$

$$Y = 3B(1 - 2\nu)$$

$$\eta = \frac{3BY}{9B - Y}$$

$$\nu = \frac{3B - 2\eta}{6B + 2\eta}$$

**\* Common Pitfall**

- ▶  $\nu > 0.5$  violates thermodynamics (impossible)
- ▶  $\nu = 0.5$ : no volume change on stretching
- ▶ If  $\nu = 0$ : no lateral strain (cork bottle cap)

**5. THERMAL STRESS & WIRE FORMULAS****Thermal stress** (rod fixed at ends):

$$\text{Thermal stress} = Y\alpha\Delta T$$

$$\text{Thermal force} = YA\alpha\Delta T$$



$\alpha$  = coefficient of linear expansion

**Elongation under own weight:**

$$\Delta L = \frac{\rho g L^2}{2Y} = \frac{WL}{2AY}$$

$W$  = weight of wire,  $\rho$  = density

**Work done in stretching wire:**

$$W = \frac{1}{2}F \cdot \Delta L = \frac{YA(\Delta L)^2}{2L}$$

\* Common Pitfall

- ▶ Thermal stress exists only when expansion is **prevented**; free expansion  $\Rightarrow$  no stress
- ▶ For hanging wire under own weight, the *maximum* stress is at the top (not middle or bottom)

## 6. FLUID PRESSURE & PASCAL'S LAW

$$P = \frac{F}{A} \text{ [Pa = N/m}^2\text{]}$$

$$P = P_0 + \rho gh \text{ (at depth } h\text{)}$$

$$P_{\text{atm}} = 1.013 \times 10^5 \text{ Pa} = 76 \text{ cm Hg} \approx 10^5 \text{ Pa}$$

**Pascal's Law:** Pressure applied to enclosed fluid is transmitted equally in all directions.

**Hydraulic Press:**

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow F_2 = F_1 \frac{A_2}{A_1}$$

**Gauge pressure:**  $P_{\text{gauge}} = P - P_{\text{atm}} = \rho gh$

**Absolute pressure:**  $P_{\text{abs}} = P_{\text{atm}} + \rho gh$

\* Common Pitfall

- ▶ Pressure is a scalar; force is a vector
- ▶  $P$  at same horizontal level is equal in connected fluids (even with different cross-sections)
- ▶ Gauge pressure can be negative (vacuum)

## 7. ARCHIMEDES' PRINCIPLE & BUOYANCY

$$\text{Buoyant force: } F_b = \rho_{\text{fluid}} \cdot V_{\text{submerged}} \cdot g$$

**Floating condition:**  $\rho_{\text{body}} \leq \rho_{\text{fluid}}$

**Fraction submerged:**

$$\frac{V_{\text{sub}}}{V_{\text{total}}} = \frac{\rho_{\text{body}}}{\rho_{\text{fluid}}}$$

**Apparent weight:**

$$W_{\text{app}} = W_{\text{true}} - F_b = V g(\rho_{\text{body}} - \rho_{\text{fluid}})$$

**Law of floatation:** Weight of body = Weight of fluid displaced

\* Common Pitfall

- ▶ Buoyant force depends on  $V_{\text{submerged}}$ , not on shape or mass of object
- ▶ A body floats even if  $\rho_{\text{body}} > \rho_{\text{fluid}}$  if it is hollow (effective density consideration)
- ▶ Buoyancy does not depend on depth of submersion

## 8. FLUID DYNAMICS & BERNOULLI'S THEOREM

**Equation of Continuity:**

$$A_1 v_1 = A_2 v_2 \text{ (Volume flow rate = const.)}$$

$$\dot{m} = \rho A v = \text{const}$$

**Bernoulli's Equation:**

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{const}$$

**Applications:**

**Torricelli's theorem** (hole in tank):

$$v = \sqrt{2gh}$$

**Venturimeter:**

$$v_1 = A_2 \sqrt{\frac{2\Delta P}{\rho(A_1^2 - A_2^2)}}$$

**Dynamic lift** (aerofoil):  $\Delta P = \frac{1}{2}\rho(v_2^2 - v_1^2)$

**Volume flow rate:**  $Q = Av \text{ [m}^3\text{/s]}$

\* Common Pitfall

- ▶ Bernoulli applies to **streamline flow** of **ideal** (non-viscous, incompressible) fluid
- ▶ Higher speed  $\Rightarrow$  lower pressure (not higher); counter-intuitive but correct
- ▶ Continuity:  $Av = \text{const}$  means narrow section has *higher* velocity

## 9. VISCOSITY & STOKES' LAW

**Newton's Law of Viscosity:**

$$F = \eta A \frac{dv}{dy}$$

$$\eta = \text{coefficient of viscosity [Pa} \cdot \text{s = Poise/10]}$$

$$1 \text{ Poise} = 0.1 \text{ Pa} \cdot \text{s}$$

**Stokes' Law:** (sphere of radius  $r$ , velocity  $v$ )

$$F_{\text{viscous}} = 6\pi\eta r v$$

**Terminal Velocity:** ( $F_{\text{viscous}} + F_b = W$ )

$$v_t = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

$\rho$  = density of sphere;  $\sigma$  = density of fluid

**Poiseuille's Formula** (pipe flow):

$$Q = \frac{\pi r^4 \Delta P}{8\eta L}$$

$$Q \propto r^4 \text{ (radius has huge effect!)}$$

**Reynolds Number:**



$$Re = \frac{\rho v d}{\eta}$$

$Re < 1000$ : laminar;  $Re > 2000$ : turbulent

★ Common Pitfall

- ▶  $v_t \propto r^2$ : doubling radius  $\Rightarrow 4\times$  terminal velocity
- ▶ Viscosity of liquids **decreases** with temperature; gases **increase** with temperature
- ▶ At terminal velocity: acceleration = 0; net force = 0

## 10. SURFACE TENSION & CAPILLARITY

### Surface Tension (T):

$$T = \frac{F}{l} \text{ [N/m]}$$

$$T = \frac{W}{\Delta A} \text{ (work per unit area) [J/m}^2\text{]}$$

### Excess Pressure inside curved surfaces:

$$\text{Liquid drop: } \Delta P = \frac{2T}{R}$$

$$\text{Air bubble in liquid: } \Delta P = \frac{2T}{R}$$

$$\text{Soap bubble (2 surfaces): } \Delta P = \frac{4T}{R}$$

### Capillarity (Jurin's Law):

$$h = \frac{2T \cos \theta}{\rho g r}$$

$$h \propto \frac{1}{r} \text{ (finer tube: higher rise)}$$

$\theta$  = contact angle;  $r$  = radius of tube

### Contact angle:

Water-glass:  $\theta \approx 0$  (wetting; rises)

Mercury-glass:  $\theta \approx 135$  (non-wetting; falls)

**Merging of droplets** ( $n$  drops of radius  $r \rightarrow 1$  drop of radius  $R$ ):

$$R = n^{1/3} r$$

$$\text{Energy released} = 4\pi T r^2 (n - n^{2/3})$$

$$= 4\pi T R^2 (n^{1/3} - 1)$$

★ Common Pitfall

- ▶ Soap bubble has **two** surfaces  $\Rightarrow \Delta P = 4T/R$ ; liquid drop has one surface  $\Rightarrow \Delta P = 2T/R$
- ▶ Surface tension decreases with temperature
- ▶ Capillary rise: liquid level *inside* tube is higher for water (wetting), lower for mercury (non-wetting)
- ▶ If tube is short (length  $< h$ ): liquid spreads at top, does NOT overflow

## 11. QUICK REVISION TABLE – MODULI

Modulus	Formula	For
Young's $Y$	$\frac{FL}{A\Delta L}$	Solids only
Bulk $B$	$-\frac{P}{\Delta V/V}$	Solids, liquids, gases
Rigidity $\eta$	$\frac{F/A}{\theta}$	Solids only

General order:  $Y > B > \eta$  (for most metals)

Formula	Use
$P = P_0 + \rho g h$	Fluid pressure
$F_b = \rho_f V_s g$	Buoyancy
$v = \sqrt{2gh}$	Torricelli
$v_t = \frac{2r^2(\rho - \sigma)g}{9\eta}$	Terminal vel.
$h = \frac{2T \cos \theta}{\rho g r}$	Capillarity
$\Delta P = 4T/R$	Soap bubble
$\Delta P = 2T/R$	Liquid drop
$Q = \frac{\pi r^4 \Delta P}{8\eta L}$	Poiseuille

## 12. MASTER: COMMON MISTAKES NEET

- Soap vs. drop:** Soap bubble =  $4T/R$  (two surfaces); liquid drop =  $2T/R$  (one surface)
- Terminal velocity:**  $v_t \propto r^2$ , not  $r$ ; doubling radius gives  $4\times v_t$
- Bernoulli:** High speed  $\Rightarrow$  low pressure (NOT high pressure)
- Elastic moduli:**  $Y$  and  $\eta$  only for solids;  $B$  for all states
- Capillarity:** Rise  $\propto 1/r$ ; narrow tube gives higher rise
- Viscosity vs. temperature:** Liquids  $\downarrow$ ; gases  $\uparrow$
- Floating:**  $F_b = W_{\text{body}}$  (not  $F_b = W_{\text{displaced}}$  for partly submerged)
- Pascal's law:** Pressure, not force, is equal throughout;  $F_2/F_1 = A_2/A_1$
- Poisson's ratio:**  $\nu \leq 0.5$  always;  $\nu = 0.5$  means incompressible
- Stress-strain:** Breaking stress (E)  $<$  Ultimate tensile strength (D) for ductile material